Using edgefx to compute the effects of habitat edges on a landscape

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1 Background

The edges of habitat patches affect species that live within the habitat. Often, edge effects are modeled as a simple function of distance to the nearest edge. Edge structure can be much more complex than that single-valued characterization, however. This package allows computation of edge effects due to all (or just nearby) edges in a landscape.

> library(edgefx)

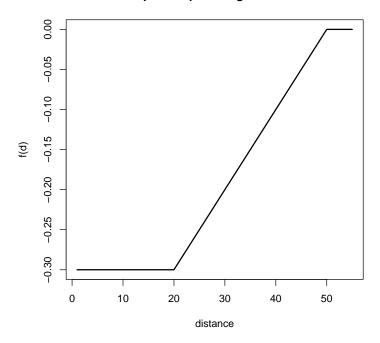
Consider a response variable describing something about a species across space. For example, this could be the density of individuals, or the height of plants. Call it z. Say there is a baseline value far from any edges, k, and that edge effects cause deviations (either positive or negative) from this. Consider a particular point on the landscape where z is or could be measured; call it the "focal" point. Consider a second point that lies along a habitat edge and is a distance d from the focal point; call it the edge point. The effect of the edge point on the focal point can be modeled simply as a "plateau point edge effect,"

$$f(d) = \begin{cases} e_0 & d \le D_0 \\ e_0 \left(1 + \frac{D_0 - d}{D_{max} - D_0} \right) & D_0 < d \le D_{max} \\ 0 & d > D_{max} \end{cases}$$
 (1)

where e_0 characterizes the maximum effect, D_0 is the distance out to which the maximum effect is felt, and D_{max} is the maximum distance at which any effect is felt. To see what f(d) looks like, use point.edge.effect():

```
> params <- list(e0 = -0.3, Dmax = 50, D0 = 20)
> d <- seq(params$Dmax * 1.1)
> plot(d, sapply(d, point.edge.effect, params), type = "1", lwd = 2,
+ xlab = "distance", ylab = "f(d)", main = "plateau point edge effect")
```

plateau point edge effect



The simplest way of dealing with edges is to consider only the distance to the nearest edge, d_{min} , for each focal point [at position (x, y)], yielding

$$z(x,y) = k + f(d_{min}). (2)$$

But this ignores the effects of other edge points, many of which may also be nearby. More complete would be to sum over all edge points within distance D_{max} , Γ , yielding

$$z(x,y) = k + \int_{\Gamma} f(s) \, ds. \tag{3}$$

There are functions in edgefx to evaluate Eq.~3 for idealized infinite-extent edges, vector-based descriptions of finite habitat edges, and gridded habitat maps. Many of these procedures also allow estimation of the parameters in Eq.~1 from such data.

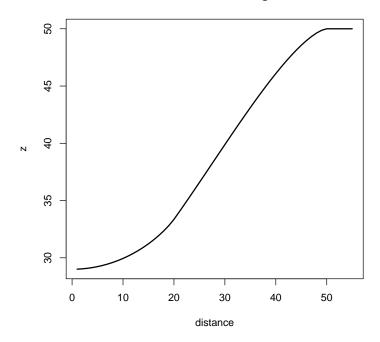
2 Infinite edges

2.1 Known parameter values

As an idealized case, consider an edge that is linear and infinite in extent. To evaluate Eq.~3 at a range of distances from the edge, using the plateau point edge effect shown above, use infinite.edge.effect():

```
> params$k = 50
> plot(d, sapply(d, infinite.edge.effect, params), type = "l",
+ lwd = 2, xlab = "distance", ylab = "z", main = "effect of infinite edge")
```

effect of infinite edge



2.2 Parameter estimation

Suppose you have edges in your landscape that approximate linear, infinite edges (or they don't really, but you want to make that assumption for comparison purposes). If you have observed values of your response variable, z, at a variety of distances from infinite edges, d, you can fit for the parameters in the point edge effect function (Eq.~1).

To see this in action, first generate some fake data:

```
> params <- list(e0 = -0.3, Dmax = 70, D0 = 20, k = 50)

> d \leftarrow seq(0, 100, 5)

> set.seed(3)

> z \leftarrow sapply(d, infinite.edge.effect, params) + rnorm(length(d))
```

Now use non-linear least squares to fit the infinite edge function, with and without the D_0 parameter:

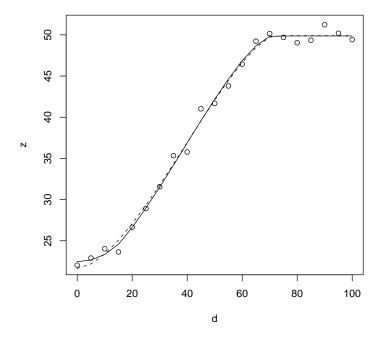
```
> nls.4par <- nls(z \sim sapply(d, infinite.edge.effect, e0, Dmax, + k, D0), start = list(e0 = -0.5, Dmax = 100, D0 = 40, k = 70), + algorithm = "port", lower = list(e0 = -Inf, Dmax = 0, k = -Inf, D0 = 0)) > <math>nls.3par <- nls(z \sim sapply(d, infinite.edge.effect, e0, Dmax, + k), start = list(e0 = -0.5, Dmax = 100, k = 70), algorithm = "port", + lower = list(e0 = -Inf, Dmax = 0, k = -Inf)) > summary(nls.4par)
```

Formula: z ~ sapply(d, infinite.edge.effect, e0, Dmax, k, D0)

Parameters:

Estimate Std. Error t value Pr(>|t|)

```
e0 -0.31595 0.01976 -15.991 1.12e-11 ***
Dmax 70.66101 1.43764 49.151 < 2e-16 ***
DO 16.09796 4.58907 3.508 0.0027 **
    49.86647 0.31088 160.405 < 2e-16 ***
k
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
Residual standard error: 0.8008 on 17 degrees of freedom
Algorithm "port", convergence message: relative convergence (4)
> summary(nls.3par)
Formula: z ~ sapply(d, infinite.edge.effect, e0, Dmax, k)
Parameters:
     Estimate Std. Error t value Pr(>|t|)
e0 -0.394812 0.009778 -40.38 <2e-16 ***
Dmax 71.571193 1.334010 53.65 <2e-16 ***
  49.905507 0.327551 152.36 <2e-16 ***
k
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
Residual standard error: 0.8362 on 18 degrees of freedom
Algorithm "port", convergence message: relative convergence (4)
To see the fits:
> plot(d, z)
> lines(d, predict(nls.4par))
> lines(d, predict(nls.3par), lty = 2)
```



To perform an AIC test of the two models:

> AIC(nls.3par) - AIC(nls.4par)

[1] 1.014882

The four-parameter model fits better (lower AIC score), but not substantially so (difference is less than 2). (But be a bit careful about potential bugginess in the nls methods of AIC and logLik.)

See Sec. 3.2.1 for notes on the nls options and convergence messages. It also shows a different method of fitting infinite edges using fake maps.

3 Vectorized landscapes

A real habitat won't consist solely of an infinite linear edge, but it can be approximated as a collection of finite edge line segments. Once you've turned the habitat edges in your landscape into line segments, you can use those edge descriptions along with observations of the response variable z(x,y) to (1) estimate the parameters in the point edge effect function, e_0 , k, D_0 , and D_{max} , and (2) use parameter values (obtained through estimation or elsehow) to predict z(x,y) anywhere in the landscape.

3.1 Input files

You should have a single input file for each focal point, containing the coordinates of the focal point and all the relevant edge segments. This package does not provide facilities for automatic identification of edges from general maps. Leslie will have notes on how best to get appropriate input files from GIS. But the per-focal-point files allow better customization of exactly which edge segments are "visible" to each focal point (e.g. don't include edges that are hidden behind other edges).

Here is an example of an input file:

The origin of the coordinate system is arbitrary. The first non-comment line gives the x and y coordinates of the focal point; it needs two extra commas so R doesn't freak out about mis-matched numbers of columns. The rest of the lines are for the endpoints of each edge segment. The first edge extends from (0, 0) to (0, 100), the second from (0, 0) to (0, 40), etc. The last column is for notes about the habitat types, or whatever you want. Actually, you can have as many trailing columns as you want; their contents are ignored so far.

We've included a collection of simple edge input files for use here. You will have to identify where on your system they were installed. For me it is /usr/local/lib/R/site-library/edgefx/doc/inputfiles/, but for the compilation of this documentation, it is just inputfiles/. You should define prefix to be wherever you find them. Let's read the example files into a list.

```
> prefix <- "inputfiles/"
> filenames <- c("edge1a.dat", "edge1b.dat", "edge1c.dat", "edge1d.dat",
+          "edge1e.dat", "edge2a.dat", "edge2b.dat", "edge2c.dat", "edge2d.dat",
+          "edge2e.dat")
> edgefilenames <- paste(prefix, filenames, sep = "")
> edgelist <- lapply(edgefilenames, read.table, sep = ",")
> names(edgelist) <- filenames</pre>
```

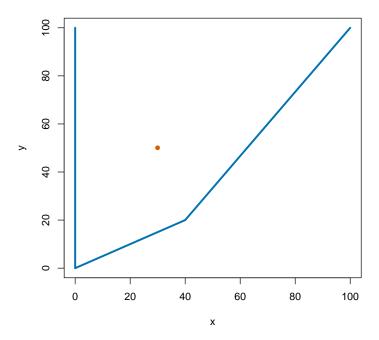
Our edgelist now has ten elements, one for each file. Here's what the first one looks like:

> edgelist[[1]]

```
V1 V2
                          ۷5
        VЗ
             ۷4
1 30 50
         NA
            NA
                    habitat1
  0
     0
          0 100
                    habitat2
3
  0 0
        40
             20
                    habitat2
4 40 20 100 100
                    habitat2
```

- > draw.edges(edgelist[[1]])
- > title(names(edgelist)[[1]])

edge1a.dat



The focal point is shown in orange, and its edges are shown in blue.

In our example, the first five input files all have the same edges but different focal points, and the same for the second five. You can combine analysis for whatever files you want, provided that you expect (or are willing to assume) that the same parameter values apply to all of them.

3.2 Parameter estimation

Now suppose that at each of the focal points, we have a measure of the response variable z. Let's read in those values.

```
> z.obs
      map
                 z
   edge1a 83.95431
2
   edge1c 72.98649
   edge1e 82.36007
3
   edge2b 92.20720
4
5
   edge2d 87.98545
   edge1b 69.62626
6
7
   edge1d 74.63733
   edge2a 78.00661
   edge2c 84.14898
10 edge2e 86.71185
```

> z.obs <- read.table(paste(prefix, "edgez.dat", sep = ""), header = T)

Described next are two possible approaches for obtaining parameter estimates and their uncertainties from these data.

3.2.1 Non-linear least squares, via nls()

To estimate the parameter values, assuming normally-distributed errors, we can do a nonlinear least squares fit to the plateau point edge function integrated over all the edges for each focal point. So our dependent variable is z.obs\$z and our independent variables come from the application of Eq.~3 to each matrix in edgelist. We need to provide a rough guess of the parameter values in order for nls() to get started.

```
> guess <- list(e0 = -0.1, Dmax = 80, k = 50, D0 = 5)
> edgefit <- edge.nls(edgelist, z.obs$z, guess)</pre>
```

You can pass additional arguments to nls() after guess, e.g., trace=T. To see how the fit did, use the result as you would any nls object, e.g.,

```
> summary(edgefit)
Formula: observed ~ by(edges[, 2:4], xvals, map.edge.effect, e0, Dmax,
   k, D0) [unique(xvals)]
Parameters:
    Estimate Std. Error t value Pr(>|t|)
e0
     -0.27611 0.02547 -10.839 3.65e-05 ***
Dmax 42.47617
                3.34888 12.684 1.47e-05 ***
k
    97.20525
                1.67003 58.206 1.73e-09 ***
                          4.894 0.00273 **
DO
     16.64404
                3.40071
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' '1
Residual standard error: 1.692 on 6 degrees of freedom
Algorithm "port", convergence message: relative convergence (4)
> library(MASS)
> edgefit.profile <- profile(edgefit)
> confint(edgefit.profile)
         2.5%
                    97.5%
e0
           NA
              -0.2143116
           NA 49.8745073
Dmax
k
    93.46509 101.3459460
```

Unfortunately, the port algorithm is required in order to constrain D_0 and D_{max} to be non-negative (this is applied within edge.nls), but port is "unfinished" (according to the nls help page). One consequence is that the profile and confint functions don't work reliably with nls results when port is used. But you'll have the standard errors from summary, even if confint gives NAs.

Support for a non-zero value of D_0 is pretty strong in this example, but often it isn't. To fit Malcolm's original point edge effect function rather than the plateau function, just omit D_0 :

```
> guess <- list(e0 = -0.1, Dmax = 80, k = 50)
> edgefit <- edge.nls(edgelist, z.obs$z, guess)
> summary(edgefit)
```

MΔ

DΩ

MΔ

```
Formula: observed ~ by(edges[, 2:4], xvals, map.edge.effect, e0, Dmax,
   k) [unique(xvals)]
Parameters:
    Estimate Std. Error t value Pr(>|t|)
                 0.0415 -9.027 4.18e-05 ***
     -0.3746
e0
                 3.4590 12.741 4.25e-06 ***
Dmax 44.0713
      96.5783
                 1.9805 48.764 3.99e-10 ***
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
Residual standard error: 1.942 on 7 degrees of freedom
Algorithm "port", convergence message: relative convergence (4)
> edgefit.profile <- profile(edgefit)
> confint(edgefit.profile)
          2.5%
                     97.5%
e0
     -0.4700749 -0.2862819
Dmax 38.4377327 51.4741232
    92.4350263 101.3855908
```

If you get a convergence message of anything other than (0) from nls (like the (4) we got here), it's best to try a variety of values for guess.

The function edge.nls() just provides a convenient wrapper to nls(). In case you want to tweak nls's options yourself (or maybe try something else, like nls2), here are the extra steps to take. You can omit DO from formula and guess if you want. (Note: I don't know why xvals and z have to be defined separately, but R has a fit if they aren't.)

An alternative method for fitting to the infinite edge function (see Sec.~2.2) is to use the above procedure (or the one below, with optim or MCMC) but to replace the data frame of map information, edges above, with values that represent infinite edges, inf.edges here:

```
3     map3 10 -Inf Inf
4     map4 15 -Inf Inf
5     map5 20 -Inf Inf
6     map6 25 -Inf Inf
```

3.2.2 Generalized likelihood optimization and MCMC

An alternative to constrained optimization with nls is to deal with the (log)likelihood of the data directly, which the function edge.lnL() provides. For Gaussian errors, the log-likelihood is proportional to the sum-of-squared-differences, so using a general-purpose optimizer like optim() with edge.lnL() is in principle the same as using nls(), though you can specify different algorithms and ways to constrain parameter values. For Poisson errors, the likelihood function is computed slightly differently, but it can be used in the same way.

To use optim() directly (rather than through a wrapper as for edge.nls()) to obtain maximum likelihood parameter estimates, we have to abide by its rules. Specifically, the initial guess must be a vector (which can be obtained from a list by unlist(); if instead its elements are unnamed, they must be in the order e_0 , D_{max} , k, and optionally D_0) and the value is minimized so the negative log-likelihood must be used (obtained from edge.lnL() with neg=TRUE). Additionally, it's a bit faster to pass optim the relocated edge dataframe (obtained via relocate.edge.df()) rather than the raw edge list.

Here is a sequence of examples:

```
> guess <- unlist(guess)
> edges <- relocate.edge.df(edgelist)
> optim(guess, edge.lnL, NULL, edges, z.obs$z, neg = T)
$par
                                                  D0
          e0
                     Dmax
                                      k
  -0.3554509 1171.5757913 170.1661713
                                          53.6657135
$value
[1] 367.7956
$counts
function gradient
     501
$convergence
[1] 1
$message
NULL
```

Consult the optim() documentation to see that a convergence value of 1 indicates that the iteration limit has been reached. We can tell optim() to try for longer:

```
$counts
function gradient
507 NA
```

\$convergence

Γ17 0

\$message

NULL

The convergence value of 0 now indicates that the optimization was successful. The maximum-likelihood parameter estimates are given by \$par, but these values are quite different from those from the nls() fit. We can find a higher likelihood (lower \$value) by using different starting values:

\$convergence

[1] 0

\$message

NULL

Now the agreement with the nls result is perfect.

When an optimization method is not specified, the default is Nelder-Mead and the parameter constraints are taken care of by returning a likelihood value of -Inf when $D_{max} < 0$ or $D_0 < 0$. You could instead use the quasi-Newton method with box constraints on the parameter values (see the optim() documentation, and be sure that the order in lower matches that in guess):

```
$convergence
[1] 0
$message
[1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"
  To fix D_0 = 0, simply omit it from the guess:
> guess < c(e0 = -0.3, Dmax = 50, k = 100)
> optim(guess, edge.lnL, NULL, edges, z.obs$z, neg = T)
$par
        e0
                 Dmax
-0.3746541 44.0709961 96.5786155
$value
[1] 26.39620
$counts
function gradient
    140
$convergence
[1] 0
```

71

\$message
NULL

NULL

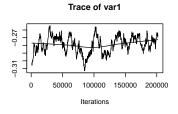
71

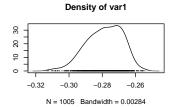
Everything above also applies to Poisson errors, which you can request with family="poisson". Note that your observed values must be positive integers—this should be intrinsically true for real data, but it must be forced in this illustration:

Poisson errors often don't work well with method="L-BFGS-B" because negative predicted values must return a likelihood of -Inf, which this method apparantly can't handle.

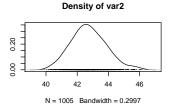
Unlike nls(), optim() does not produce a structure for use by functions like confint. You could still map out the likelihood surface to get confidence intervals, but I couldn't find general-purpose R functions for that.

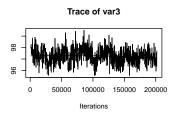
Alternatively, issues like uncertainty and correlation in the parameter estimates can be addressed by obtaining posterior distributions from Markov chain Monte Carlo. You will have to learn about proper use of MCMC elsewhere, including diagnosing convergence, but here is an example to start with. (It could also take family="poisson", presumably.) The initial values are informed by the optim() results, to reduce the burn-in time. The tuning values were chosen by experimentation to yield an acceptance probability of about 20%. The thinning interval was chosen after looking at autocorrelation plots (acf() is useful for this). It takes awhile to run.

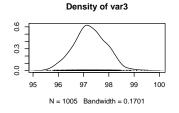


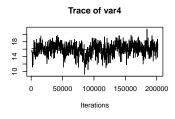


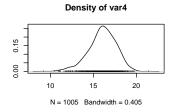
Trace of var2 9 9 0 50000 100000 150000 200000 Iterations











> summary(edgemcmc)

Iterations = 1001:201801
Thinning interval = 200
Number of chains = 1
Sample size per chain = 1005

1. Empirical mean and standard deviation for each variable, plus standard error of the mean:

 Mean
 SD
 Naive SE
 Time-series SE

 [1,] -0.2788 0.01068 0.0003368
 0.001690

 [2,] 42.7434 1.12665 0.0355390
 0.042792

 [3,] 97.2555 0.63961 0.0201759
 0.042251

 [4,] 16.0457 1.60456 0.0506142
 0.114726

2. Quantiles for each variable:

2.5% 25% 50% 75% 97.5% var1 -0.3015 -0.2861 -0.2781 -0.2704 -0.2618

```
var2 40.6454 41.9654 42.6749 43.4879 45.2359
var3 95.9890 96.8341 97.2294 97.6966 98.4394
var4 12.3598 15.1368 16.1176 17.1772 18.8182
```

4 Gridded landscapes

As mentioned above, edgefx does not have sophisticated edge-detection functions. For a gridded landscape, the most it will do is identify as edges the cells of non-habitat ("matrix," but I'll avoid that term since it is also an R data structure) that have, among their four neighboring cells, at least one habitat cell. The response value z at each cell in the landscape can then be predicted as the sum of effects from all identified edge cells.

The distance between two cells is defined to be 1 for adjacent cells, and can be found for any pair of cell coordinates like so:

```
> distance(c(2, 3), c(4, 7))
[1] 4.472136
```

If you have existing parameter estimates and want to apply them here, you may have to do a unit conversion. The exact conversion will depend on what you have, but here is an example. Say your observations, z, are the number of individuals per grid cell of size length a. And say that when you estimated the edge function parameters (e_0, D_{max}, k) , and maybe D_0 , you gave distances in meters rather than number of grid cells. Since D_{max} and D_0 have units of length, the adjusted values you should use with the unit grid cells here are $D'_{max} = D_{max}/a$ and $D'_0 = D_0/a$. Since k already has units of per grid cell, k' = k. Since e_0 has units of individuals per length, $e'_0 = e_0 \times a$.

4.1 Input files

Here is an example of an input file for a gridded landscape. Note that 0 signifies non-habitat, any other positive number signifies habitat, and spaces signify borders between cells (this allows habitat codes of more than one digit).

Let's read that particular habitat file into a matrix:

```
> map <- read.table(paste(prefix, "smallgrid.dat", sep = ""))
> map <- data.matrix(map)</pre>
```

I found it easier to visualize the landscape with slightly different formatting. The function write.grid() produces a file, whose contents are printed below. You can find the example files near the prefix you defined above (replace inputfiles with outputfiles).

The next step is to identify the cells that are edges. This can be slightly slow for a large landscape. You may also want to write those results to a file for visualization/checking.

```
> edges <- find.edges(map)
> write.edges(map, edges, "outputfiles/smallgrid.edge")
```

creates a file that contains:

4.2 Response prediction

To compute the response value for each cell, we first need to provide parameter values for the plateau point edge effect function.

```
> params <- list(e0 = -0.3, k = 5, D0 = 0, Dmax = 10)
```

Then we can use grid.effects() to treat each habitat cell in turn as the focal cell and compute its response value, z. This step can be slow for a large grid and large values of D_{max} .

```
> z <- grid.effects(map, edges, params)
```

The result is a vector with one item per cell, ordered by column (read down column 1, then read down column 2, etc.). These values are the predicted responses for habitat cells, and NA for non-habitat cells. If you didn't want to include some cells identified as edges, you could just remove them from edges before calling grid.effects().

To print the results in either tabular or graphic form, you have to take some care to get the orientation of the landscape right. For example, here is how to view z in the same orientation as the map:

> matrix(z, ncol = ncol(map))

```
[,1]
                   [,2]
                             [,3]
                                       [,4]
                                                 [,5]
                                                           [,6] [,7]
                                                                     [,8]
                                                                           [,9] [,10]
[1,]
           NA
                     NA
                               NA
                                         NA
                                                   NA
                                                             NA
                                                                  NA
                                                                        NA
                                                                             NA
                                                                                    NA
[2,] 2.757224 2.391091 2.064658 1.761261 1.509462
                                                                  NA
                                                                        NA
                                                                             NA
                                                                                    NA
                                                             NA
[3,] 2.778329 2.406429 2.046681 1.702641
                                                   NA
                                                             NA
                                                                  NA
                                                                        NA
                                                                             NA
                                                                                    NA
[4,] 2.867153 2.501380 2.121457
                                                   NA
                                         NA
                                                             NA
                                                                  NA
                                                                        NΑ
                                                                             NA
                                                                                    ΝA
[5,] 3.019753 2.676498 2.302272
                                         NA
                                                   NA
                                                             NA
                                                                  NA
                                                                        NA
                                                                             NA
                                                                                    NA
[6,] 3.221152 2.925479 2.582877 2.249200 1.953873 1.756154
                                                                  NA
                                                                        NA
                                                                             NA
                                                                                    ΝA
        [,11]
                  [,12]
                            [,13]
                                      [,14]
[1,] 2.005296 2.304855 2.648967 2.989033
[2,]
           NA 2.016763 2.371094 2.747569
[3,] 1.555564 1.826897 2.168306 2.565966
[4,]
                               NA
           NA
                     NA
                                         NΑ
[5,]
           NA
                     NA
                               NA
                                         NA
[6,]
           NA
                     NA 2.388755 2.727175
```

And here is how to write a file for z that has the same cell layout as the input file.

```
> write.table(matrix(z, nrow = nrow(map)), file = "outputfiles/smallgrid.z",
+ na = "NA", quote = F, sep = " ", row.names = F, col.names = F)
```

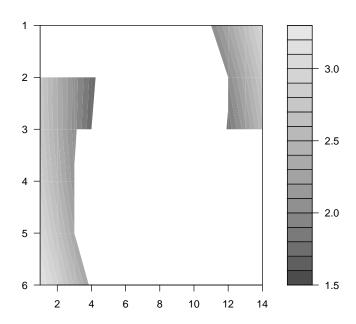
If your input map had a variety of habitat codes and you want to extract the values of z that go with each, you can do it like this.

```
> m <- as.vector(as.matrix(map))
> hcodes <- list(one = 1, two = 2)
> sapply(hcodes, f <- function(x) z[m == x])

$one
  [1] 2.757224 2.778329 2.867153 3.019753 3.221152 2.391091 2.406429 2.501380
  [9] 2.676498 2.925479 2.064658 2.046681 2.121457 2.302272 2.582877 1.761261
[17] 1.702641 2.249200 1.509462 1.953873 1.756154 2.388755 2.727175

$two
  [1] 2.005296 1.555564 2.304855 2.016763 1.826897 2.648967 2.371094 2.168306
[9] 2.989033 2.747569 2.565966

To plot the results, turn z into a matrix in what seems like the wrong orientation:
> z <- matrix(z, byrow = T, ncol = nrow(map))
Here's a basic plot of the result.
> filled.contour(seq(ncol(map)), seq(nrow(map)), z, ylim = c(nrow(map),
```

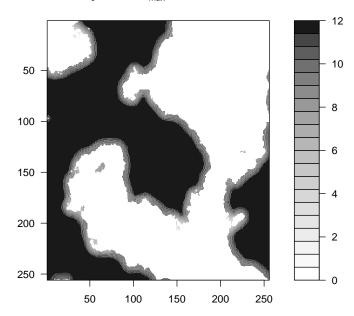


1), color.palette = gray.colors)

(You might instead want to use contourplot from the lattice package, which uses a clearer formula notation.)

That's not such an exciting landscape. Here's a more elaborate one (the interior areas are black, but it seems that gets lost in the pdf conversion):





If the computations are slow and you only want responses predicted for a portion of your landscape, you can create a dataframe of focal cell coordinates and just use those. For example, to predict on just a strip in the upper left corner:

```
> focals <- data.frame(row = rep(seq(5), 2), col = c(rep(1, 5),
      rep(2, 5))
> focals
   row col
1
     1
         1
2
     2
         1
3
     3
         1
4
         1
5
     5
         1
6
     1
         2
7
```

```
8 3 2
9 4 2
10 5 2
```

> $z \leftarrow apply(focals, 1, grid.edge.effect, edges, map, params)$